Surface acoustic waves and the magnetoconductivity of a two-dimensional electron gas

1996 J. Phys.: Condens. Matter 8 6597

(http://iopscience.iop.org/0953-8984/8/36/012)

J. Phys.: Condens. Matter **8** (1996) 6597–6605. Printed in the UK

Surface acoustic waves and the magnetoconductivity of a two-dimensional electron gas

M R Eskildsen† and H Smith‡

† Departament of Solid State Physics, Risø National Laboratory, PO Box 49, DK-4000 Roskilde, Denmark
‡ Ørsted Laboratory, H C Ørsted Institute, Universitetsparken 5, DK-2100 København Ø, Denmark

Received 9 May 1996

Abstract. The frequency- and wavevector-dependent conductivity tensor of a two-dimensional electron gas is determined within a semiclassical model containing two different characteristic times, a transport time and a lifetime. The velocity shift of a surface acoustic wave, caused by the piezoelectric coupling, is calculated as a function of magnetic field. The results are applicable under conditions where the frequency of the surface acoustic wave is comparable to the cyclotron frequency. The calculated velocity shift is compared to that observed for composite fermions in the quantum Hall regime near half-filling.

1. Introduction

The two-dimensional electron gas (2DEG) in a magnetic field exhibits a wealth of fascinating phenomena that reflect the subtle nature of the electron motion in strong magnetic fields, giving rise to both the integer and the fractional quantum Hall effect. In recent years there has been increasing experimental evidence [1–6] of the existence of a new kind of quasiparticle, a composite fermion consisting of an electron with two flux quanta attached [7, 8]. With the fractional quantum Hall effect being observed at filling factors ν equal to $\frac{1}{3}$, $\frac{2}{5}$, $\frac{3}{7}$ etc, the filling factor $\nu = \frac{1}{2}$ represents the limit from below (as well as from above) of the hierarchy of filling fractions yielding quantized Hall resistance, but the half-filled Landau level is not in itself a quantum Hall state.

The composite-fermion picture, as developed by Halperin, Lee and Read [8] (HLR), is derived from a Chern–Simons transformation applied to the interacting electron gas in a magnetic field corresponding to the filling factor $\nu = \frac{1}{2}$. As a result of this transformation the system may be described approximately in terms of independent quasiparticles, 'composite fermions', moving in a magnetic field of zero average. The theory predicts the existence of a Fermi surface at exactly half-filling, for which there is now experimental support [1, 3, 6]. At the mean-field level of this description one may use semiclassical transport theory, based on the Boltzmann equation, for calculating the 2DEG conductivity near the filling factor of one half. Since measurements of the velocity shift of a surface acoustic wave (SAW) furnish direct information on the conductivity of the 2DEG, it has become possible to make a detailed comparison between theory and experiment. A semiclassical calculation of the conductivity was carried out by HLR by treating the collision integral in the relaxation time approximation involving a single lifetime τ.

0953-8984/96/366597+09$19.50

The aim of the present paper is to extend previous calculations of the velocity shift in two respects, both of experimental importance: (1) we allow for the possibility that the transport time, τ_{tr}, differs significantly from the lifetime, τ; and (2) we consider situations where the frequency of the SAW is comparable to the cyclotron frequency and compare our results to those obtained previously in the static limit.

A large difference between the transport time and the lifetime is a characteristic of two-dimensional electron gases, in which the mobility is limited by scattering from ionized impurities that are well separated from the two-dimensional layer. For electrons moving in low magnetic fields the transport time is up to two orders of magnitude larger than the lifetime [9]. The implications of this for the Shubnikov–de Haas oscillations were discussed by Coleridge *et al* [10]. For composite fermions, the resistivity measurements by Leadley *et al* [4] suggest that τ_{tr} is about one order of magnitude larger than τ. We therefore calculate the magnetoconductivity within a model containing two different characteristic times, a transport time and a lifetime. Our finite-frequency calculations are relevant to the interpretation of the recent high-frequency SAW measurements by Willett *et al* [6].

The results that we obtain are applicable both to electrons moving in a weak magnetic field, and to the quantum Hall regime with composite fermions moving in a weak *effective* magnetic field.

In the following section we solve the semiclassical transport equation and derive a general expression for the conductivity as a function of the magnetic field and the frequency of the SAW. In section 3 we compare our calculated velocity shift of a SAW to that observed experimentally in the quantum Hall regime near half-filling. A conclusion is given in section 4.

2. The semiclassical transport equation

The system that we shall consider is one in which the 2DEG is confined to the xy-plane, with a magnetic field in the z-direction, and with an electrical field propagating in the direction of the x-axis, $\boldsymbol{E} \propto \mathrm{e}^{\mathrm{i}(qx-\omega t)}$.

2.1. Solution of the Boltzmann equation

The aim of the following is to solve the Boltzmann equation

$$\frac{\partial f}{\partial t} + \boldsymbol{v}\cdot\boldsymbol{\nabla} f + \dot{\boldsymbol{k}}\cdot\boldsymbol{\nabla}_{\boldsymbol{k}} f = \left(\frac{\partial f}{\partial t}\right)_{\mathrm{Coll}}. \tag{1}$$

We introduce the deviation function, ψ, via the conventional definition $f = f_0 - \psi\,\partial f_0/\partial\varepsilon$, where f_0 is the equilibrium Fermi distribution function. The force term is written as $\hbar\dot{\boldsymbol{k}} = -e\boldsymbol{E} - e\boldsymbol{v}\times\boldsymbol{B}$. Expressing $\boldsymbol{k}$ and $\boldsymbol{v}$ in polar coordinates results in

$$-\frac{e}{\hbar}\boldsymbol{v}\times\boldsymbol{B}\cdot\boldsymbol{\nabla}_{\boldsymbol{k}}\psi = \omega_c\frac{\partial\psi}{\partial\varphi}$$

where $\omega_c = eB/m^*$ is the cyclotron frequency, and m^* is the electron effective mass, while φ is the angle between $\boldsymbol{k}$ (or $\boldsymbol{v}$) and the x-axis. We separate out the derivative of the equilibrium distribution function via the definition

$$\left(\frac{\partial f}{\partial t}\right)_{\mathrm{Coll}} = -I(\psi)\frac{\partial f_0}{\partial\varepsilon}$$

with the functional $I(\psi)$ to be specified below. As ψ has the same space and time dependence as the electrical field, i.e. $\psi \propto e^{i(qx-\omega t)}$, the Boltzmann equation becomes

$$\omega_c \frac{\partial \psi}{\partial \varphi} - i(\omega - qv_F \cos\varphi)\psi - I(\psi) = -ev_F(E_x \cos\varphi + E_y \sin\varphi) \tag{2}$$

since $\boldsymbol{q}$ is along the x-axis. The longitudinal electrical field (E_x) originates in the piezoelectricity of the medium, while the transverse electrical field (E_y) is due to charge build-up at the edges of the 2DEG.

Now the substitution $\psi = \psi' e^{-iX\sin\varphi}$, with $X = qv_F/\omega_c$, is performed in order to eliminate the $qv_F \cos\varphi$-term in equation (2). Expressing ψ' by the series

$$\psi' = \sum_{n=-\infty}^{\infty} a_n e^{in\varphi} \tag{3}$$

and multiplying by $(2\pi)^{-1}e^{-im\varphi}$ and integrating over φ from zero to 2π one finds

$$i(m\omega_c - \omega)a_m - \int_0^{2\pi} \frac{d\varphi}{2\pi}\, I(\psi)e^{-im\varphi + iX\sin\varphi} = -ev_F\left(E_x \frac{m}{X} - iE_y \frac{\partial}{\partial X}\right) J_m(X) \tag{4}$$

where the Js denote Bessel functions.

To proceed further it is necessary to specify the form of the collision term, $I(\psi)$. Let us first follow HLR and consider the simplest approximation involving a single relaxation time, τ:

$$I(\psi) = -\frac{\psi}{\tau}. \tag{5}$$

This leads to

$$(1 + i(m\omega_c - \omega)\tau)a_m = -ev_F\tau\left(E_x \frac{m}{X} - iE_y \frac{\partial}{\partial X}\right) J_m(X). \tag{6}$$

The current density is given by

$$\boldsymbol{j} = -\sum_\sigma e \int \frac{d^2k}{(2\pi)^2}\, \boldsymbol{v} f.$$

Inserting the expression for the distribution function one finds

$$\boldsymbol{j} = -\frac{n_e e}{\pi m^* v_F} \sum_{n=-\infty}^{\infty} a_n \int_0^{2\pi} d\varphi \begin{pmatrix} \cos\varphi \\ \sin\varphi \end{pmatrix} e^{in\varphi - iX\sin\varphi}.$$

Here the summation over spin indices has been carried out, yielding the electron density $n_e = k_F^2/2\pi$. Written out in components the current density is

$$j_x = -\frac{2n_e e}{m^* v_F} \sum_{n=-\infty}^{\infty} a_n \frac{n}{X} J_n(X) \tag{7a}$$

$$j_y = -\frac{2n_e e}{m^* v_F} \sum_{n=-\infty}^{\infty} a_n i \frac{\partial J_n(X)}{\partial X}. \tag{7b}$$

When the value for a_n found from equation (6) is inserted, we obtain

$$\sigma_{xx}^0 = 2\sigma_0' \sum_{n=-\infty}^{\infty} \frac{1}{1 + i(n\omega_c - \omega)\tau} \left(\frac{n}{X} J_n(X)\right)^2 \tag{8a}$$

$$\sigma_{xy}^0 = -2i\sigma_0' \sum_{n=-\infty}^{\infty} \frac{1}{1 + i(n\omega_c - \omega)\tau} \frac{n}{X} J_n(X) \frac{\partial J_n(X)}{\partial X} \tag{8b}$$

$$\sigma_{yy}^0 = 2\sigma_0' \sum_{n=-\infty}^{\infty} \frac{1}{1 + i(n\omega_c - \omega)\tau} \left(\frac{\partial J_n(X)}{\partial X}\right)^2 \tag{8c}$$

with $\sigma_0' = n_e e^2 \tau / m^*$. The static limit ($\omega = 0$) of equations (8*a*)–(8*c*) was used by Willett *et al* [1, 6] for comparison with the measured velocity shift of a surface acoustic wave.

2.2. Extending the relaxation time approximation

We now wish to consider the experimentally important case wherein the transport time differs from the lifetime due, e.g., to the predominance of small-angle scattering. We therefore consider an extended relaxation time approximation, with a collision term of the form

$$I(\psi) = -\frac{1}{\tau}\left(\psi - (\psi, \cos\varphi)\lambda\cos\varphi - (\psi, \sin\varphi)\lambda\sin\varphi\right) \tag{9}$$

where τ and λ are constants. In this expression for the collision term, $(\psi, \cos\varphi)$ is the inner product

$$(\psi, \cos\varphi) = \int_0^{2\pi} \frac{z\ \mathrm{d}\varphi}{\pi}\ \psi\ \cos\varphi$$

and similarly for $(\psi, \sin\varphi)$. It should be noted that the solution (6) yields a non-vanishing value of

$$\langle\psi\rangle = \int_0^{2\pi} \frac{\mathrm{d}\varphi}{2\pi}\psi$$

but since $\langle\psi\rangle$ is seen to be odd in q there is no net change in density when the contributions from q and $-q$ are added. The use of a simple relaxation time approximation is thus consistent with the requirements of particle number conservation, $\langle\psi\rangle = 0$, and it is therefore not necessary to ensure particle conservation by explicitly subtracting $\langle\psi\rangle$ in equation (9).

The collision term (9) has two eigenvalues $1/\tau$ and $(1-\lambda)/\tau$. The latter is associated with the eigenfunctions $\cos\varphi$ and $\sin\varphi$, and equals the inverse transport time entering the d.c. conductivity, as we shall see below.

The Boltzmann equation now has the following form:

$$H\psi = -ev_F\tau E\cos\varphi + \lambda\cos\varphi(\psi, \cos\varphi) + \lambda\sin\varphi(\psi, \sin\varphi) \tag{10}$$

in the case where the electric field points along the x-direction and

$$H\psi = -ev_F\tau E\sin\varphi + \lambda\cos\varphi(\psi, \cos\varphi) + \lambda\sin\varphi(\psi, \sin\varphi) \tag{11}$$

when the electric field points along the y-direction. Here H is the differential operator given by

$$H = 1 - \mathrm{i}\omega\tau + \omega_c\tau\frac{\partial}{\partial\phi} + \mathrm{i}qv_F\tau\cos\phi. \tag{12}$$

The conductivity tensor σ_{ij}^0 given by (8*a*)–(8*c*) above is obtained when λ in equations (10) and (11) is set equal to zero. Since $-ev_F\tau H^{-1}\cos\varphi$ is the solution to the Boltzmann equation (10) with $\lambda = 0$, one has $\sigma_{xx}^0 = \sigma_0'(H^{-1}\cos\varphi, \cos\varphi)$ and similarly for the other components.

Now let us consider the case where λ differs from zero. For convenience we derive the conductivity in units of σ_0'. In the absence of the sound wave and the magnetic field, the conductivity is thus given by $\sigma_{xx} = \sigma_{yy} = 1/(1-\lambda)$. By inverting the operator H we then find that the elements of the (dimensionless) conductivity tensor must satisfy the following equations:

$$
\begin{aligned}
\sigma_{xx} &= \sigma^0_{xx} + \lambda\sigma^0_{xx}\sigma_{xx} + \lambda\sigma^0_{xy}\sigma_{yx} \\
\sigma_{yx} &= \sigma^0_{yx} + \lambda\sigma^0_{yx}\sigma_{xx} + \lambda\sigma^0_{yy}\sigma_{yx} \\
\sigma_{xy} &= \sigma^0_{xy} + \lambda\sigma^0_{xy}\sigma_{yy} + \lambda\sigma^0_{xx}\sigma_{xy} \\
\sigma_{yy} &= \sigma^0_{yy} + \lambda\sigma^0_{yy}\sigma_{yy} + \lambda\sigma^0_{yx}\sigma_{xy}.
\end{aligned}
$$

It is straightforward to solve these four equations for the four elements of the conductivity tensor, in terms of those corresponding to $\lambda = 0$. The result is conveniently expressed in terms of the trace (tr) and determinant (det) of the conductivity tensor, σ^0, obtained for $\lambda = 0$. One gets:

$$\sigma_{xx} = \frac{\sigma^0_{xx} - \lambda \det \sigma^0}{1 - \lambda \operatorname{tr}\sigma^0 + \lambda^2 \det \sigma^0} \tag{13a}$$

$$\sigma_{xy} = \frac{\sigma^0_{xy}}{1 - \lambda \operatorname{tr}\sigma^0 + \lambda^2 \det \sigma^0} \tag{13b}$$

$$\sigma_{yy} = \frac{\sigma^0_{yy} - \lambda \det \sigma^0}{1 - \lambda \operatorname{tr}\sigma^0 + \lambda^2 \det \sigma^0}. \tag{13c}$$

From the expressions given in (8*a*)–(8*c*) and (13*a*)–(13*c*) we can recover various limiting forms. Consider first the static, homogeneous case, $\omega = q = 0$. In this case $J_n(X) = \delta_{n,0}$, and one finds the d.c. conductivity

$$\sigma_{\mathrm{dc}} = \frac{\sigma_0}{1 + (\omega_c\tau_{\mathrm{tr}})^2} \begin{pmatrix} 1 & -\omega_c\tau_{\mathrm{tr}} \\ \omega_c\tau_{\mathrm{tr}} & 1 \end{pmatrix} \tag{14}$$

where the transport time is given by $\tau_{\mathrm{tr}} = \tau/(1 - \lambda)$, while $\sigma_0 = n_e e^2 \tau_{\mathrm{tr}}/m^*$. Similarly, for $\omega \neq 0$ while $q = B = 0$, we recover the Drude expression for the real and imaginary part of the conductivity.

For finite q and ω and $B = 0$, the expressions (13*a*) and (13*c*) reduce to

$$\sigma_{xx} = \frac{F(\beta)}{1 - \lambda F(\beta)} \tag{15}$$

for the xx-component, while the yy-component is

$$\sigma_{yy} = \frac{G(\beta)}{1 - \lambda G(\beta)} \tag{16}$$

with $\beta = qv_F\tau/(1 - \mathrm{i}\omega\tau)$. The functions $F(\beta)$ and $G(\beta)$ are given by

$$F(\beta) = \frac{1}{1 - \mathrm{i}\omega\tau}\,\frac{2}{\beta^2}\left(1 - \frac{1}{\sqrt{1 + \beta^2}}\right) \tag{17}$$

and

$$G(\beta) = \sqrt{1 + \beta^2}\;F(\beta) \tag{18}$$

respectively. Again it is easily seen that equations (15) to (18) reduce to the Drude expression if q is set equal to zero.

3. The velocity shift of a surface acoustic wave

The expressions (13*a*) to (13*c*) for the 2DEG conductivity tensor are applicable both to the case of electrons moving in low magnetic fields and to the case of composite fermions in the quantum Hall regime. In this section we shall compare our calculated velocity shift to SAW measurements in the quantum Hall regime near half-filling.

For composite fermions the applied magnetic field is substituted for with an *effective* magnetic field given by

$$\Delta B = B - B_{1/2} \tag{19}$$

where $B_{1/2} = 2n_e\phi_0$ is the magnetic field corresponding to exactly half-filling of the lowest Landau level, and ϕ_0 is the flux quantum h/e. The 2DEG is assumed to be completely spin polarized by the applied magnetic field so that $n_e = k_F^2/4\pi$. The 2DEG resistivity tensor is given by [8]

$$\rho = \tilde{\rho} + \rho_{\text{cs}} \tag{20}$$

where $\tilde{\rho}$ is the so-called intrinsic composite-fermion resistivity tensor which is equal to the inverse of the conductivity tensor given by equations (13*a*) to (13*c*), and ρ_{cs} is

$$\rho_{\text{cs}} = 2\frac{h}{e^2}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \tag{21}$$

This leads to

$$\sigma_{xx} = \frac{\tilde{\rho}_{yy}(q,\omega)}{(\rho_{\text{cs}})_{xy}^2}$$

as $(\rho_{\text{cs}})_{xy} \gg \tilde{\rho}_{yy}, \tilde{\rho}_{yy}$.

The velocity shift of the SAW depends on the 2DEG conductivity through the coupling of the piezoelectrically generated electrical field. This leads to a velocity shift of the SAW reflecting the 2DEG conductivity, [11, 12]:

$$\frac{\Delta v_s}{v_s} = \frac{\alpha^2}{2}\text{Re}\left[\frac{1}{1 + \mathrm{i}\sigma_{xx}/\sigma_m}\right] \tag{22}$$

where $\alpha^2/2$ is the piezoelectric coupling constant and σ_m lies in the range between $(\epsilon + \epsilon_0)v_s$ and $2\epsilon v_s$. A derivation of formula (22) for the velocity shift is given in appendix A.

The parameter values used in the following are similar to those of Willett *et al* [6]. The mean free path is taken to be $l = 0.5\ \mu$m, the electron density is $n_e = 1.6 \times 10^{11}$ cm^{-2}, the composite-fermion effective mass is $m^* = 0.8m_e$, and the background conductivity is $\sigma_0 = 1/2300\ \Omega$. This yields a transport time given by $\tau_{\text{tr}} = 24.2$ ps. The SAW wavelength and frequency are $\lambda_{\text{SAW}} = 2200$ Å and $f_{\text{SAW}} = 10.7$ GHz respectively. The resulting dimensionless parameters are $ql = 12$ and $\omega\tau_{\text{tr}} = 1.6$. In the expression for the velocity shift we use the piezoelectric coupling, $\alpha^2/2 = 3.2 \times 10^{-4}$, and $\sigma_m = 35 \times 10^{-7}\ \Omega$. As discussed by Willett *et al* [6] the value of σ_m is used as a fitting parameter and differs somewhat from theoretical expectation.

First we compare the resonances obtained in the static limit ($\omega = 0$) with the effect of including the finite frequency in the calculation. In figure 1 we show the calculated velocity shift for $ql = 12$ with $\omega\tau_{\text{tr}}$ equal to 0 and 1.6. For simplicity we take $\lambda = 0$ corresponding to $\tau = \tau_{\text{tr}}$. The most significant feature of the two curves is that the position of the principal resonance at $\Delta B \sim 0.6$ T is only slightly shifted despite the fact that $\omega = 0.5\omega_c$ at this value of the effective field. In addition two other features are noteworthy: (1) a phase shift of π in the field region with ΔB between 0.2 and 0.4 T and (2) a shoulder at $\Delta B = 0.43$ T.

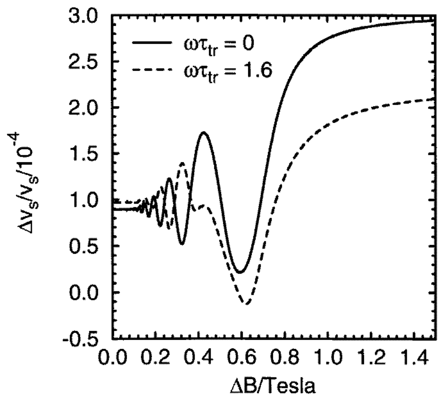

Figure 1. Velocity shift versus effective magnetic field near half-filling of the lowest Landau level. The two curves represent the static limit ($\omega = 0$) and the effect of a finite frequency ($\omega\tau_{tr} = 1.6$ corresponding to $\omega/2\pi = 10.7$ GHz).

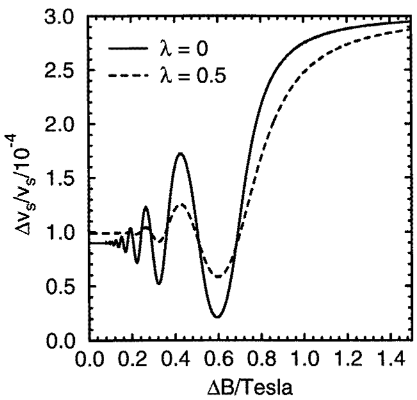

Figure 2. SAW velocity shift versus effective magnetic field for the cases $\tau = \tau_{tr}$ ($\lambda = 0$) and $\tau = \tau_{tr}/2$ ($\lambda = 0.5$).

Looking at the resonance positions in the velocity shift we find a shift of the principal resonance from $\Delta B = 0.59$ to 0.62 T, and a shift of the secondary resonance from 0.32 to 0.27 T, the latter arising from the above-mentioned phase shift.

In figure 2 we show the velocity shift in the static limit with $ql = 12$ and for λ equal to 0 and 0.5 corresponding to $\tau = \tau_{tr}$ and $\tau = \tau_{tr}/2$ respectively. We observe a suppression of the amplitude of the oscillations in the velocity shift as λ is increased, while the position of the resonances is unchanged. This is the behaviour that we would qualitatively expect. The limiting parameter for the number and amplitude of the observed oscillations is $\omega_c\tau$, and an increase of λ is, assuming a constant mean free path and thus a constant τ_{tr}, effectively a reduction of $\omega_c\tau$. The opposite effect, i.e. an increase in the number and amplitude of the oscillations, is obtained if one increases τ_{tr} keeping the ratio τ_{tr}/τ constant. If λ was taken to be as large as 0.9 corresponding to $\tau = \tau_{tr}/10$ the oscillations would be completely suppressed for the parameters chosen.

4. Conclusion

We have calculated the conductivity of a two-dimensional electron gas from solutions to the semiclassical Boltzmann equation, taking into account the effects of a finite frequency and allowing for a difference between the lifetime, τ, and the transport time, τ_{tr}. The resulting expression for the conductivity tensor is generally applicable and may be used to determine the velocity shift of a surface acoustic wave under different physical conditions, such as those of electrons moving in weak magnetic fields or composite fermions in the quantum Hall regime near half-filling.

In the case of composite fermions we find that the principal resonance in the velocity shift is only slightly shifted when the SAW frequency is as large as half the cyclotron frequency at the position of the principal resonance, while more pronounced differences occur at lower values of the effective magnetic field. When the transport time is larger than the lifetime, the amplitude of the oscillations in the velocity shift are decreased while the resonance positions remain unchanged.

Acknowledgment

We would like to thank M A Paalanen for a very helpful conversation.

Appendix A. Velocity shift

The derivation given here is based on a simplified model of the system, but yields results that are quantitatively reliable. The formula for the velocity shift is derived for a bulk wave moving in the x-direction in a piezoelectric medium containing a 2DEG in the xy-plane at z equal to zero, following Hutson and White [13].

The equations connecting the mechanical stress, T, and the electrical displacement, D, with the mechanical strain, S, and the electrical field, E, are

$$\begin{aligned} T &= cS - eE_x \\ D_x &= eS + \epsilon E_x \\ D_z &= \epsilon E_z. \end{aligned}$$

Here c is the stiffness constant giving the wave velocity in the absence of the 2DEG, e is the piezoelectric coupling constant and ϵ is the dielectric permittivity of the piezoelectric material.

The electric potential is considered as consisting of two parts: one part from the wave and one part from the induced charge-density fluctuations in the 2DEG:

$$\begin{aligned} &\phi = \phi_{\mathrm{W}} + \phi_{\mathrm{2DEG}} \\ &\phi_{\mathrm{W}} = A\mathrm{e}^{\mathrm{i}(kx-\omega t)} \\ &\phi_{\mathrm{2DEG}} = B_1\mathrm{e}^{\mathrm{i}(kx-\omega t)-kz} \qquad \text{for } z > 0 \\ &\phi_{\mathrm{2DEG}} = B_2\mathrm{e}^{\mathrm{i}(kx-\omega t)+kz} \qquad \text{for } z < 0. \end{aligned}$$

The constants are determined from the boundary conditions. For $|z| \to \infty$ the disappearance of the divergence of the displacement leads to

$$A = -\frac{e}{\epsilon k^2}\frac{\partial S}{\partial x}\mathrm{e}^{-\mathrm{i}(kx-\omega t)}.$$

At z equal to zero, the continuity of the longitudinal part of the electrical field across the 2DEG implies that $B_1 = B_2$ $(= B)$. Also at z equal to zero the change in the transverse part of the displacement is given by the surface charge density, $\rho_\square$:

$$D_{1z} - D_{2z} = 2\epsilon k B\mathrm{e}^{\mathrm{i}(kx-\omega t)} = \rho_\square.$$

Differentiating this expression with respect to time, and utilizing the continuity equation

$$\frac{\partial \rho_\square}{\partial t} = -\boldsymbol{\nabla}\cdot\boldsymbol{j} = -\sigma_{xx}\frac{\partial E_x}{\partial x}$$

one finds

$$-2\mathrm{i}\omega\epsilon k B\mathrm{e}^{\mathrm{i}(kx-\omega t)} = \sigma_{xx}\frac{\partial E_x}{\partial x}.$$

This leads to the following relation between A and B:

$$B = -\frac{1}{1 - 2\mathrm{i}\epsilon v_s/\sigma_{xx}}A$$

with $v_s = \omega/k$.

The longitudinal electrical field at $z = 0$ is then given by

$$\frac{\partial E_x}{\partial x} = -\frac{e}{\epsilon}\frac{1}{1 + \mathrm{i}\sigma_{xx}/(2\epsilon v_s)}\frac{\partial S}{\partial x}$$

leading to

$$\frac{\partial T}{\partial x} = c\left(1 + \frac{e^2}{c\epsilon}\frac{1}{1 + \mathrm{i}\sigma_{xx}/(2\epsilon v_s)}\right)\frac{\partial S}{\partial x} = c'\frac{\partial S}{\partial x}.$$

The relative velocity shift is thus

$$\frac{\Delta v_s}{v_s} = \mathrm{Re}\left(\sqrt{\frac{c'}{c}}\right) - 1 = \frac{\alpha^2}{2}\mathrm{Re}\left[\frac{1}{1 + \mathrm{i}\sigma_{xx}/\sigma_m}\right] \quad (1)$$

with the coupling constant $\alpha^2 = e^2/c\epsilon$ depending on the distance of the 2DEG from the surface and $\sigma_m = 2\epsilon v_s$.

If the 2DEG is situated at the surface, the expression for σ_m should be modified to $\sigma_m = (\epsilon + \epsilon_0)v_s$, where ϵ_0 is the vacuum permittivity. If the 2DEG is sufficiently near the surface, σ_m/v_s assumes a value intermediate between $\epsilon + \epsilon_0$ and 2ϵ.

References

[1] Willett R L, Ruel R R, Paalanen M A, West K W and Pfeiffer L N 1993 *Phys. Rev.* B **47** 7344
Willett R L, Ruel R R, West K W and Pfeiffer L N 1993 *Phys. Rev. Lett.* **71** 3846

[2] Du R R, Störmer H L, Tsui D C, Pfeiffer L N and West K W 1993 *Phys. Rev. Lett.* **70** 2944; 1994 *Solid State Commun.* **90** 71
Du R R, Störmer H L, Tsui D C, Yeh A S, Pfeiffer L N and West K W 1994 *Phys. Rev. Lett.* **73** 3274

[3] Kang W, Störmer H L, Pfeiffer L N, Baldwin K W and West K W 1993 *Phys. Rev. Lett.* **71** 3850

[4] Leadley D R, Nicholas R J, Foxon C T and Harris J J 1994 *Phys. Rev. Lett.* **72** 1906

[5] Manoharan H C, Shayegan M and Kleper S J 1994 *Phys. Rev. Lett.* **73** 3270

[6] Willett R L, West K W and Pfeiffer L N 1995 *Phys. Rev. Lett.* **75** 2988

[7] Jain J K 1989 *Phys. Rev. Lett.* **63** 199; 1989 *Phys. Rev.* B **40** 8079; 1990 *Phys. Rev.* B **41** 7653

[8] Halperin B I, Lee P A and Read N 1993 *Phys. Rev.* B **47** 7312

[9] Paalanen M A, Tsui D C and Hwang J C M 1983 *Phys. Rev. Lett.* **51** 2226 (erratum: 1984 **52** 484)

[10] Coleridge P T, Stoner R and Fletcher R 1989 *Phys. Rev.* B **39** 1120

[11] Efros A L and Galperin Y M 1990 *Phys. Rev. Lett.* **64** 1959

[12] The formula for the velocity shift is usually quoted from Efros and Galperin [11] although the formula for the velocity shift is not stated explicitly. Note that the expressions for σ_m quoted in reference [3] and in reference [11] differ by a factor of two. As discussed in appendix A the numerical factor in σ_m depends on the position of the 2DEG with respect to the surface.

[13] Hutson A R and White D L 1962 *J. Appl. Phys.* **33** 40